

Nuclear constraints on non-Newtonian gravity at femtometer scale

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Effects of the non-Newtonian gravity on properties of finite nuclei are studied by consistently incorporating both the direct and exchange contribution of the Yukawa potential in the Hartree-Fock approach using a well-tested Skyrme force for the strong interaction. It is shown for the first time that the strength of the Yukawa term in the non-Newtonian gravity is limited to $\log(|\alpha|) < 1.75/[\lambda(\text{fm})]^{0.54} + 33.6$ within the length scale of $\lambda = 1 - 10$ fm in order for the calculated properties of finite nuclei not to be in conflict with accurate experimental data available.

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The quest to unify gravity with other fundamental forces is among the most challenging scientific questions for the new century [1]. While it is generally assumed that non-relativistic gravity obeys Newton's Inverse-Square-Law (ISL) for all distances greater than the Plank length of about 1.6×10^{-33} cm [2–4], the Newtonian gravity has been tested and various upper limits [5–7] on the ISL violation have been set down to only about 10 fm so far [8]. In fact, motivated by the possible existence of both extra dimensions within string/M theories and new particles in the supersymmetric extension of the Standard Model, it has long been proposed that the Newtonian gravitational potential between two objects of masses m_1 and m_2 at positions \vec{r}_1 and \vec{r}_2 may be modified to [9]

$$V_{\text{grav}}(\vec{r}_1, \vec{r}_2) = -\frac{Gm_1m_2}{|\vec{r}_1 - \vec{r}_2|}(1 + \alpha e^{-|\vec{r}_1 - \vec{r}_2|/\lambda}), \quad (1)$$

where G is the gravitational constant, α and λ are the strength and length scale of the non-Newtonian Yukawa potential, respectively. Moreover, several modified gravity theories including the scalar-tensor-vector gravity [10] and $f(R)$ gravity [11] lead to such a non-Newtonian potential at the weak-field limit. While the main purpose of this work is to constrain the parameters of the Yukawa term at femtometer scales, it is important to emphasize that this kind of non-Newtonian potentials have been tested extensively against well-known observations at galaxy scales, see, e.g., Ref. [12] for a recent review. In particular, we notice here that the non-Newtonian potential of Eq. (1) has been used to successfully explain both the flattening of the galaxy rotation curves away from the Kepler limit, see, e.g. Refs. [13, 14] and the Bullet Cluster 1E0657-558 observations in the absence of dark matter [15]. Moreover, it was shown that, unlike for massive

particles, the motion of massless particles is not affected by the Yukawa term. Consequently, all the lensing observables obtained with the non-Newtonian potential of Eq. (1) are equal to the ones known from General Relativity thanks to suitable cancelations in the post-Newtonian limit [16, 17]. Thus, to explain the gravitational lensing phenomenon, dark matter seems still needed with the non-Newtonian potential considered here.

In the one-boson-exchange picture, the Yukawa potential may come from exchanging a light and weakly coupled spin-0 axion [18] or spin-1 U -boson [19] corresponding respectively to an attractive or repulsive potential, and it can be further written as

$$V_Y(\vec{r}_1, \vec{r}_2) = \pm \frac{g^2}{4\pi} \frac{e^{-\mu|\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|}, \quad (2)$$

where $\mu = 1/\lambda$ is the boson mass, and $g = \sqrt{4\pi|\alpha|Gm^2}$ is the boson-nucleon coupling constant with m being the nucleon mass. The exchanged boson may mediate the annihilation of dark matter particles [20, 21] and it is related to several interesting new phenomena in particle physics and cosmology [22, 23]. It was also found recently that the Yukawa term affects significantly the Equation of State (EOS) of neutron-rich nuclear matter, thus properties of neutron stars (NSs), e.g., their mass-radius relation, moment of inertia and the core-crust transition density/pressure [24–27]. While observed properties of NSs can be very well reproduced by combining the Yukawa term using appropriate parameters with nuclear EOSs that otherwise failed to do so by themselves [24, 25], the Yukawa term is not absolutely necessary because of the poorly known nature of the EOS of dense neutron-rich nuclear matter. On the other hand, it is not ruled out either. To our best knowledge, no quantitative constraint exists on the strength of the Yukawa term at femtometer or shorter length scale.

Nuclei are testing grounds of fundamental interactions among nucleons in the femtometer range. Generally speaking, experimental measurements of global properties of stable nuclei, such as their binding energies and charge radii, are very well reproduced consistently

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within about 2% uncertainty using established nuclear many-body theories and our current knowledge about the strong, weak and Coulomb forces without considering gravity. Moreover, these properties are most strongly influenced by the well known isoscalar part of the nuclear strong interaction around the saturation density of nuclear matter. Thus, in order to be consistent with current theoretical understanding and experimental observations in nuclear physics, effects of gravity on properties of stable nuclei have to be less than about 2%. Using this requirement and by consistently incorporating both the direct and exchange contributions of gravity in the Hartree-Fock (HF) approach using a well established Skyrme force for the strong interaction, we show for the first time that the strength of the Yukawa term is limited to $\log(|\alpha|) < 1.75/[\lambda(\text{fm})]^{0.54} + 33.6$ in the range of $\lambda = 1 - 10$ fm. This provides a reliable reference to test stringently non-Newtonian gravitational theories in this previously unexplored region.

The contribution from the non-Newtonian Yukawa potential to the potential energy of the nuclear system is

$$E_Y = \frac{1}{2} \sum_{i,j} \langle ij | (1 - P_r P_\sigma P_\tau) V_Y | ij \rangle, \quad (3)$$

where P_r , P_σ , and P_τ are the space, spin, and isospin exchange operator, respectively, and $|ij\rangle$ is the quantum state of the i th particle containing spatial, spin, and isospin parts. The first term in Eq. (3) is the direct contribution and can be calculated from

$$E_Y^D = \frac{1}{2} \int \rho(\vec{r}_1) \rho(\vec{r}_2) \frac{g^2}{4\pi} \frac{e^{-\mu|\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|} d^3 r_1 d^3 r_2, \quad (4)$$

where $\rho(\vec{r}) = \sum_{i,\sigma,\tau} \phi_{\tau i}^*(\vec{r}, \sigma) \phi_{\tau i}(\vec{r}, \sigma)$ is the nucleon density with $\phi_{\tau i}(\vec{r}, \sigma)$ being the spatial wave function of the i th particle with spin σ and isospin τ . Using $P_\sigma = (1 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)/2$, with $\vec{\sigma}_{1(2)}$ being the Pauli operator acting on the first (second) term, the second term in Eq. (3) representing the exchange contribution can be expressed as

$$E_Y^E = -\frac{1}{4} \sum_{\tau=n,p} \int [\rho_\tau(\vec{r}_1, \vec{r}_2) \rho_\tau(\vec{r}_2, \vec{r}_1) + \vec{\rho}_\tau(\vec{r}_1, \vec{r}_2) \cdot \vec{\rho}_\tau(\vec{r}_2, \vec{r}_1)] \frac{g^2}{4\pi} \frac{e^{-\mu|\vec{r}_1 - \vec{r}_2|}}{|\vec{r}_1 - \vec{r}_2|} d^3 r_1 d^3 r_2, \quad (5)$$

where $\rho_\tau(\vec{r}_1, \vec{r}_2) = \sum_{i,\sigma} \phi_{\tau i}^*(\vec{r}_1, \sigma) \phi_{\tau i}(\vec{r}_2, \sigma)$ and $\vec{\rho}_\tau(\vec{r}_1, \vec{r}_2) = \sum_i \sum_{\sigma,\sigma'} \phi_{\tau i}^*(\vec{r}_1, \sigma') \phi_{\tau i}(\vec{r}_2, \sigma) \langle \sigma' | \vec{\sigma} | \sigma \rangle$ are the off-diagonal scalar and vector part of the density-matrix, respectively. Introducing the coordinate transformation $\vec{r} = (\vec{r}_1 + \vec{r}_2)/2$ and $\vec{s} = \vec{r}_1 - \vec{r}_2$, they can be calculated from the density-matrix expansion

method [28, 29]

$$\begin{aligned} & \rho_\tau(\vec{r} + \frac{\vec{s}}{2}, \vec{r} - \frac{\vec{s}}{2}) \rho_\tau(\vec{r} - \frac{\vec{s}}{2}, \vec{r} + \frac{\vec{s}}{2}) \\ & \approx \rho_\tau^2(\vec{r}) \rho_{SL}^2(k_\tau s) + 2\rho_\tau(\vec{r}) \rho_{SL}(k_\tau s) g(k_\tau s) s^2 \\ & \times \left[\frac{1}{4} \nabla^2 \rho_\tau(\vec{r}) - \tau_\tau(\vec{r}) + \frac{3}{5} k_\tau^2 \rho_\tau(\vec{r}) \right], \end{aligned} \quad (6)$$

$$\vec{\rho}_\tau(\vec{r} + \frac{\vec{s}}{2}, \vec{r} - \frac{\vec{s}}{2}) \approx \frac{i}{2} j_0(k_\tau s) \vec{s} \times \vec{J}_\tau(\vec{r}). \quad (7)$$

In the above, $k_\tau = (3\pi^2 \rho_\tau)^{1/3}$ is the Fermi momentum, $\tau_\tau(\vec{r}) = \sum_{i,\sigma} |\nabla \phi_{\tau i}(\vec{r}, \sigma)|^2$ is the kinetic energy density, and $\vec{J}_\tau(\vec{r}) = -i \sum_i \sum_{\sigma,\sigma'} \phi_{\tau i}^*(\vec{r}, \sigma') \nabla \phi_{\tau i}(\vec{r}, \sigma) \times \langle \sigma' | \vec{\sigma} | \sigma \rangle$ is the spin density. The $\rho_{SL}(k_\tau s)$ and $g(k_\tau s)$ can be expressed in terms of the first- and the third-order spherical Bessel function as $\rho_{SL}(k_\tau s) = 3j_1(k_\tau s)/(k_\tau s)$ and $g(k_\tau s) = 35j_3(k_\tau s)/[2(k_\tau s)^3]$, respectively.

Using the expressions from the density-matrix expansion, the potential energy density from the exchange contribution of the Yukawa potential can thus be written as

$$\begin{aligned} H_Y^E(\vec{r}) &= \sum_{\tau=n,p} \{ A[\rho_\tau(\vec{r})] + B[\rho_\tau(\vec{r})] \tau_\tau(\vec{r}) \\ &+ C[\rho_\tau(\vec{r})] [\nabla \rho_\tau(\vec{r})]^2 + \varphi[\rho_\tau(\vec{r})] J_\tau^2(\vec{r}) \}, \end{aligned} \quad (8)$$

where

$$\begin{aligned} A(\rho_\tau) &= -\frac{1}{4} \int \rho_\tau^2 \rho_{SL}^2(k_\tau s) V_Y(s) d^3 s \\ &- \frac{3}{5} (3\pi^2)^{2/3} \rho_\tau^{5/3} B(\rho_\tau), \end{aligned} \quad (9)$$

$$B(\rho_\tau) = \frac{1}{2} \int \rho_\tau \rho_{SL}(k_\tau s) g(k_\tau s) s^2 V_Y(s) d^3 s, \quad (10)$$

$$C(\rho_\tau) = \frac{1}{4} \frac{dB(\rho_\tau)}{d\rho_\tau}, \quad (11)$$

$$\varphi(\rho_\tau) = -\frac{\pi}{6} \int j_0^2(k_\tau s) s^4 V_Y(s) ds, \quad (12)$$

with $V_Y(s) = g^2 e^{-\mu s}/(4\pi s)$.

To calculate properties of finite nuclei within the HF approach, we solve the Schrödinger equation

$$\begin{aligned} & \left[-\nabla \cdot \frac{\hbar^2}{2m_\tau^*(\vec{r})} \nabla + U_\tau(\vec{r}) + \vec{W}_\tau(\vec{r}) \cdot (-i)(\nabla \times \vec{\sigma}) \right] \phi_{\tau i} \\ &= e_{\tau i} \phi_{\tau i} \end{aligned} \quad (13)$$

where m_τ^* , U_τ and \vec{W}_τ are the nucleon effective mass, the single-particle potential, and the form factor of the one-body spin-orbit potential, respectively. From the variational principle, the Yukawa contribution to the single-particle potential has a direct term U_τ^{YD} and an exchange

term U_τ^{YE} , namely,

$$U_\tau^{\text{Y}} = U_\tau^{\text{YD}} + U_\tau^{\text{YE}} \quad (14)$$

$$U_\tau^{\text{YD}} = \int \rho(\vec{r}') \frac{g^2}{4\pi} \frac{e^{-\mu|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} d^3r', \quad (15)$$

$$U_\tau^{\text{YE}} = \frac{dA(\rho_\tau)}{d\rho_\tau} + \frac{dB(\rho_\tau)}{d\rho_\tau} \tau_\tau - \frac{dC(\rho_\tau)}{d\rho_\tau} (\nabla \rho_\tau)^2 - 2C(\rho_\tau) \nabla^2 \rho_\tau + \frac{d\varphi(\rho_\tau)}{d\rho_\tau} J_\tau^2. \quad (16)$$

The Yukawa contribution to the form factor \vec{W}_τ^{Y} is

$$\vec{W}_\tau^{\text{Y}} = 2\varphi(\rho_\tau) \vec{J}_\tau. \quad (17)$$

In addition, the effective mass is modified by the Yukawa term according to

$$\frac{\hbar^2}{2m_\tau^*} \rightarrow \frac{\hbar^2}{2m_\tau^*} + B(\rho_\tau). \quad (18)$$

Most available nuclear effective interactions have been tuned to reproduce not only empirical properties of symmetric nuclear matter at saturation density but also global properties of finite nuclei along the β stability line. These interactions differ mostly in their isovector parts and/or the effective three-body forces used. In this work, we use properties, such as the charge radii and binding energies of stable medium to heavy nuclei that are not much affected by the still uncertain parts of the strong nuclear interaction. Specifically, we use here the MSL0 parameter set of a Skyrme-like force which has been tested extensively and was shown to describe the charge radii and binding energies of finite nuclei very well [30].

Before exploring effects of gravity on properties of finite nuclei, it is instructive to first compare the magnitudes of the Coulomb and Yukawa potentials between two protons. Shown in Fig. 1 are the Yukawa potentials using $\alpha = -1.24 \times 10^{36}$ or -1.24×10^{34} , and $\lambda = 1$ fm or 10 fm, respectively. With $\alpha = -1.24 \times 10^{36}$, which leads to the coupling constant $g^2/4\pi = 1/137$, and a long interaction range of $\lambda = 10$ fm, the Yukawa potential approaches the Coulomb potential at short distances, while it decreases faster towards long distances for smaller values of λ .

As an example illustrating effects of the Yukawa potential on properties of finite nuclei, shown in Fig. 2 are the charge density profiles of ^{208}Pb obtained within the HF approach using the MSL0 interaction with different Yukawa parameters. With α on the order of 10^{34} , the Yukawa potential has negligible effects on the charge density profile. Increasing the magnitude of α to 10^{36} , effects of the Yukawa potential become significant especially for larger values of λ .

Next we investigate more systematically how the Yukawa potential affects the charge radii r_c and binding energies (B.E.) of three typical medium to heavy nuclei by varying continuously the strength parameter α . Shown in Fig. 3 are the results for ^{208}Pb , ^{120}Sn ,

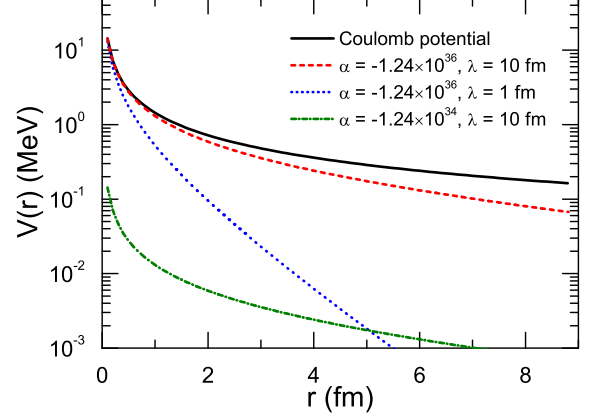


FIG. 1: (color online) A comparison of the Coulomb and Yukawa potentials between two protons as a function of separation with different parameters.

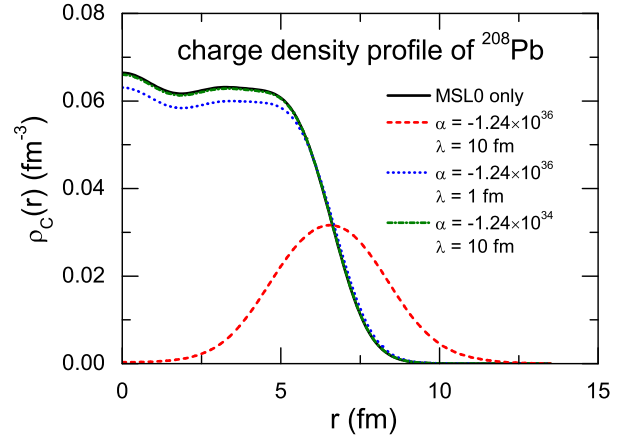


FIG. 2: (color online) Charge density profiles of ^{208}Pb from MSL0 with and without the Yukawa potential from different parameters.

and ^{48}Ca with $\lambda = 1$ fm and 10 fm, respectively. The horizontal lines are the mean values of the experimental data [31, 32]. As the Yukawa potential varies from being repulsive (with negative values for α) to attractive, the charge radius decreases and the nuclei becomes more bound. The effect is roughly proportional to the value of α . Moreover, the effects are larger for heavier nuclei due to the finite-range nature of the Yukawa potential.

As mentioned earlier, essentially all existing nuclear effective interactions can describe the experimental charge radii and binding energies of medium to heavy nuclei within about 2%. The latter is thus the largest room available to accommodate effects of gravity. As shown by the green bands in Fig. 3, this then allows us to set a nuclear upper limit on the strength of the Yukawa potential in the length range of 1 to 10 fm. Here we use the Yukawa potential effects on ^{208}Pb which is more strongly

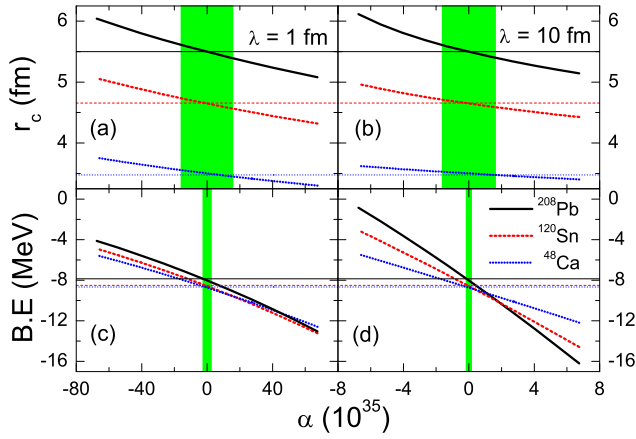


FIG. 3: (color online) Charge radius r_c [(a), (b)] and binding energy (B.E.) [(c), (d)] of ^{208}Pb , ^{120}Sn , and ^{48}Ca as functions of α at different length scales. The results at $\alpha = 0$ are those from the MSL0 interaction only. The green bands are constrained α values from taking 2% as the uncertainty of the charge radii and binding energies.

affected by gravity. More specifically, the constraint from the binding energies can be parameterized as $\log(|\alpha|) < 1.75/[\lambda(\text{fm})]^{0.54} + 33.6$ while that from the charge radii can be written as $\log(|\alpha|) < 1.18/[\lambda(\text{fm})]^{0.79} + 35.0$ for $\lambda = 1 - 10$ fm. Since the binding energy is more sensitive to the strength of Yukawa potential, the first constraint is more stringent and can be used as the nuclear upper limit. This constraint thus limits the coupling constant of the exchanged bosons with nucleons to $\log(g^2) < 0.10[\mu(\text{MeV})]^{0.54} - 3.53$ if the mass of the bosons is between 20 MeV and 200 MeV. Previously, the shortest range probed is above 10 fm in neutron scattering experiments [8, 33–36]. For a comparison, shown in Fig. 4 are the various upper limits on the magnitude of α in the range of 10^{-15} to 10^{-10} m. The constraint extracted from the present work extends to the previously unexplored region of 1 – 10 fm. Overall, there is a clear trend that the allowed deviation from Newtonian gravity increases as the interaction range decreases, reflecting the increasing difficulties of the measurement.

In summary, effects of the non-Newtonian gravity on properties of finite nuclei were studied within the Hartree-Fock approach incorporating a well-tested Skyrme force for the strong interaction and the non-Newtonian gravitational potential. For the first time, the strength of the Yukawa term is limited to $\log(|\alpha|) <$

$1.75/[\lambda(\text{fm})]^{0.54} + 33.6$ within the length range of $\lambda = 1 - 10$ fm in order for the calculated properties of finite nuclei not to be in conflict with the very accurate experimental data available. This constraint serves as a useful reference in constraining properties of weakly-coupled gauge bosons and further explorations of possible extra dimensions at femtometer scale.

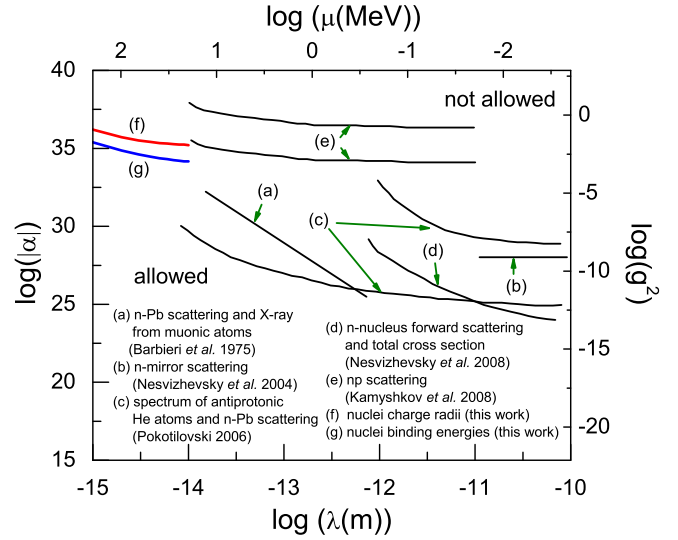


FIG. 4: (color online) The constraints of α for $\lambda = 1 - 10$ fm from charge radii and binding energies of heavy nuclei and those at longer distances extracted from analyzing neutron scattering experiments in Refs. [8, 33–36].

Acknowledgments

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